RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT

4722/01

MATHEMATICS

Core Mathematics 2

THURSDAY 7 JUNE 2007

Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 A geometric progression $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 15 \quad \text{and} \quad u_{n+1} = 0.8u_n \quad \text{for } n \geqslant 1.$$

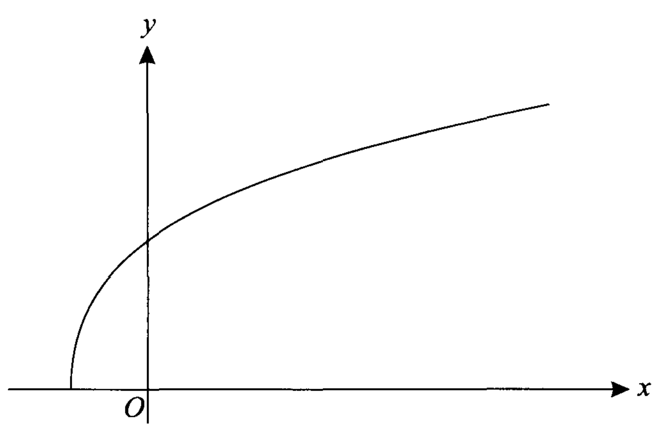
(i) Write down the values of u_2, u_3 and u_4. [2]

(ii) Find $\sum_{n=1}^{20} u_n$. [3]

2 Expand $\left(x + \frac{2}{x}\right)^4$ completely, simplifying the terms. [5]

3 Use logarithms to solve the equation $3^{2x+1} = 5^{200}$, giving the value of x correct to 3 significant figures. [5]

4



The diagram shows the curve $y = \sqrt{4x + 1}$.

(i) Use the trapezium rule, with strips of width 0.5, to find an approximate value for the area of the region bounded by the curve $y = \sqrt{4x + 1}$, the x-axis, and the lines $x = 1$ and $x = 3$. Give your answer correct to 3 significant figures. [4]

(ii) State with a reason whether this approximation is an under-estimate or an over-estimate. [2]

5 (i) Show that the equation

$$3\cos^2\theta = \sin\theta + 1$$

can be expressed in the form

$$3\sin^2\theta + \sin\theta - 2 = 0.$$ [2]

(ii) Hence solve the equation

$$3\cos^2\theta = \sin\theta + 1,$$

giving all values of θ between 0° and 360°. [5]

6 **(a)** **(i)** Find $\int x(x^2 - 4)\,dx$. [3]

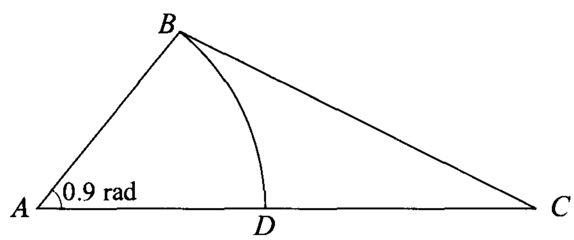
(ii) Hence evaluate $\int_1^6 x(x^2 - 4)\,dx$. [2]

(b) Find $\int \frac{6}{x^3}\,dx$. [3]

7 **(a)** In an arithmetic progression, the first term is 12 and the sum of the first 70 terms is 12 915. Find the common difference. [4]

(b) In a geometric progression, the second term is -4 and the sum to infinity is 9. Find the common ratio. [7]

8



The diagram shows a triangle ABC, where angle BAC is 0.9 radians. BAD is a sector of the circle with centre A and radius AB.

(i) The area of the sector BAD is $16.2\,\text{cm}^2$. Show that the length of AB is 6 cm. [2]

(ii) The area of triangle ABC is twice the area of sector BAD. Find the length of AC. [3]

(iii) Find the perimeter of the region BCD. [6]

9 The polynomial $f(x)$ is given by

$$f(x) = x^3 + 6x^2 + x - 4.$$

(i) **(a)** Show that $(x + 1)$ is a factor of $f(x)$. [1]

(b) Hence find the exact roots of the equation $f(x) = 0$. [6]

(ii) **(a)** Show that the equation

$$2\log_2(x+3) + \log_2 x - \log_2(4x+2) = 1$$

can be written in the form $f(x) = 0$. [5]

(b) Explain why the equation

$$2\log_2(x+3) + \log_2 x - \log_2(4x+2) = 1$$

has only one real root and state the exact value of this root. [2]

1 (i)	$u_2 = 12$	B1		State $u_2 = 12$
	$u_3 = 9.6$, $u_4 = 7.68$ (or any exact equivs)	B1√	2	Correct u_3 and u_4 from their u_2
(ii)	$S_{20} = \frac{15(1-0.8^{20})}{1-0.8}$	M1		Attempt use of $S_n = \frac{a(1-r^n)}{1-r}$, with n = 20 or 19
	$= 74.1$	A1		Obtain correct unsimplified expression
		A1	3	Obtain 74.1 or better
	OR			
		M1		List all 20 terms of GP
		A2		Obtain 74.1
			5	
2	$\left(x+\frac{2}{x}\right)^4 = x^4 + 4x^3\left(\frac{2}{x}\right) + 6x^2\left(\frac{2}{x}\right)^2 + 4x\left(\frac{2}{x}\right)^3 + \left(\frac{2}{x}\right)^4$	M1*		Attempt expansion, using powers of x and $^2/_x$ (or the two terms in their bracket), to get at least 4 terms
		M1*		Use binomial coefficients of 1, 4, 6, 4, 1
		A1dep*		Obtain two correct, simplified, terms
		A1		Obtain a further one correct, simplified, term
	$= x^4 + 8x^2 + 24 + \frac{32}{x^2} + \frac{16}{x^4}$ (or equiv)	A1	5	Obtain a fully correct, simplified, expansion
	OR			
		M1*		Attempt expansion using all four brackets
		M1*		Obtain expansion containing the correct 5 powers only (could be unsimplified powers eg $x^3 . x^{-1}$)
		A1dep*		Obtain two correct, simplified, terms
		A1		Obtain a further one correct, simplified, term
		A1		Obtain a fully correct, simplified, expansion
			5	
3	$\log 3^{(2x+1)} = \log 5^{200}$	M1		Introduce logarithms throughout
	$(2x+1)\log 3 = 200\log 5$	M1		Drop power on at least one side
		A1		Obtain correct linear equation (now containing no powers)
	$2x+1 = \frac{200\log 5}{\log 3}$	M1		Attempt solution of linear equation
	$x = 146$	A1	5	Obtain x = 146, or better
OR				
	$(2x + 1) = \log_3 5^{200}$	M1		Intoduce $\log_3$ on right-hand side
	$2x + 1 = 200\log_3 5$	M1		Drop power of 200
		A1		Obtain correct equation
		M1		Attempt solution of linear equation
		A1		Obtain x = 146, or better
			5	
4 (i)	area $\approx \frac{1}{2}\times\frac{1}{2}\times\left\{\sqrt{5}+2\left(\sqrt{7}+\sqrt{9}+\sqrt{11}\right)+\sqrt{13}\right\}$	M1		Attempt y-values for at least 4 of the x = 1, 1.5, 2, 2.5, 3 only
		M1		Attempt to use correct trapezium rule
		A1		Obtain $\frac{1}{2}\times\frac{1}{2}\times\left\{\sqrt{5}+2\left(\sqrt{7}+\sqrt{9}+\sqrt{11}\right)+\sqrt{13}\right\}$, or decimal equiv
	$\approx 0.25\times 23.766...$			
	≈ 5.94	A1	4	Obtain 5.94 or better (answer only is 0/4)
(ii)	This is an underestimate……	*B1		State underestimate
	…as the tops of the trapezia are below the curve	B1dep*B1	2	Correct statement or sketch
			6	

5	(i)	$3(1-\sin^2\theta)=\sin\theta+1$	M1		Use $\cos^2\theta = 1-\sin^2\theta$
		$3-3\sin^2\theta=\sin\theta+1$			
		$3\sin^2\theta+\sin\theta-2=0$	A1	2	Show given equation correctly
	(ii)	$(3\sin\theta-2)(\sin\theta+1)=0$	M1		Attempt to solve quadratic equation in $\sin\theta$
		$\sin\theta = \frac{2}{3}$ or -1	A1		Both values of $\sin\theta$ correct
		$\theta = 42^\circ, 138^\circ, 270^\circ$	A1		Correct answer of 270°
			A1		Correct answer of 42°
			A1√	5	For correct non-principal value answer, following their first value of θ in the required range (any extra values for θ in required range is max 4/5) (radians is max 4/5) SR: answer only (or no supporting method) is B1 for 42°, B1√ for 138°, B1 for 270°
				7	
6	(a) (i)	$\int x^3-4x = \frac{1}{4}x^4-2x^2+c$	M1		Expand and attempt integration
			A1		Obtain ¼ $x^4 - 2x^2$ (A0 if ∫ or dx still present)
			B1	3	+ c (mark can be given in (b) if not gained here)
	(ii)	$\left[\frac{1}{4}x^4-2x^2\right]_1^6$	M1		Use limits correctly in integration attempt (ie F(6) – F(1))
		=(324 – 72) – (¼ – 2) = 253¾	A1	2	Obtain 253¾ (answer only is M0A0)
	(b)	$\int 6x^{-3}\mathrm{d}x = -3x^{-2}+c$	B1		Use of $\frac{1}{x^3}=x^{-3}$
			M1		Obtain integral of the form kx^{-2}
			A1	3	Obtain correct $-3x^{-2}$ (+ c) (A0 if ∫ or dx still present, but only penalise once in question)
				8	
7	(a)	$S_{70} = \frac{70}{2}\{(2\times 12)+(70-1)d\}$	M1		Attempt S_{70}
			A1		Obtain correct unsimplified expression
		35(24 + 69d) = 12915	M1		Equate attempt at S_{70} to 12915, and attempt to find d
		d = 5	A1	4	Obtain d = 5
OR		$\frac{70}{2}\{12+l\}=12915$	M1		Attempt to find d by first equating $^n/_2\,(a + l)$ to 12915
		l = 357	A1		Obtain l = 357
		12 + 69d = 357	M1		Equate u_{70} to l
		d = 5	A1		Obtain d = 5
	(b)	ar = -4	B1		Correct statement for second term
		$\frac{a}{1-r}=9$	B1		Correct statement for sum to infinity
		$\frac{-4}{r}=9-9r$ or $a=9-\left(9\times\frac{-4}{a}\right)$	M1		Attempt to eliminate either a or r
		$9r^2-9r-4=0$ $a^2-9a-36=0$	A1		Obtain correct equation (no algebraic denominators/brackets)
		$(3r-4)(3r+1)=0$ $(a+3)(a-12)=0$	M1		Attempt solution of three term quadratic equation
		$r=\frac{4}{3}$, $r=-\frac{1}{3}$ a = -3, a = 12	A1		Obtain at least $r=-\frac{1}{3}$ (from correct working only)
		Hence $r=-\frac{1}{3}$	A1	7	Obtain $r=-\frac{1}{3}$ only (from correct working only) SR: answer only / T&I is B2 only
				11	

8 **(i)** $\frac{1}{2}\times AB^2 \times 0.9 = 16.2$	M1	Use $\left(\frac{1}{2}\right)r^2\theta = 16.2$
$AB^2 = 36 \Rightarrow AB = 6$	A1 **2**	Confirm AB = 6 cm (or verify ½ x 6^2 x 0.9 = 16.2)
(ii) $\frac{1}{2}\times 6\times AC\times \sin 0.9 = 32.4$	M1*	Use $\Delta = \frac{1}{2}bc\sin A$, or equiv
	M1dep*	Equate attempt at area to 32.4
AC = 13.8 cm	A1 **3**	Obtain AC = 13.8 cm, or better
(iii) $BC^2 = 6^2 + 13.8^2 - 2\times 6\times 13.8\times \cos 0.9$	M1	Attempt use of correct cosine formula in ΔABC
	A1√	Correct unsimplified equation, from their AC
Hence BC = 11.1 cm	A1	Obtain BC = 11.1 cm, or anything that rounds to this
$BD = 6\times 0.9 = 5.4$ cm	B1	State BD = 5.4 cm (seen anywhere in question)
Hence perimeter = 11.1 + 5.4 + (13.8 – 6)	M1	Attempt perimeter of region BCD
= 24.3 cm	A1 **6**	Obtain 24.3 cm, or anything that rounds to this
	11	
9 **(i)** **(a)** f (-1) = – 1 + 6 – 1 – 4 = 0	B1 **1**	Confirm f(-1) = 0, through any method
(b) $x = -1$	B1	State x = -1 at any point
$f(x) = (x+1)(x^2 + 5x - 4)$	M1	Attempt complete division by (x + 1), or equiv
	A1	Obtain $x^2 + 5x + k$
	A1	Obtain completely correct quotient
$x = \frac{-5 \pm \sqrt{25+16}}{2}$	M1	Attempt use of quadratic formula, or equiv, find roots
$x = \frac{1}{2}\left(-5 \pm \sqrt{41}\right)$	A1 **6**	Obtain $\frac{1}{2}\left(-5 \pm \sqrt{41}\right)$
(ii) **(a)** $\log_2(x+3)^2 + \log_2 x - \log_2(4x+2) = 1$	B1	State or imply that 2log (x + 3) = log $(x + 3)^2$
	M1	Add or subtract two, or more, of their algebraic logs correctly
$\log_2\left(\frac{(x+3)^2 x}{4x+2}\right) = 1$	A1	Obtain correct equation (or any equivalent, with single term on each side)
$\frac{(x+3)^2 x}{4x+2} = 2$	B1	Use $\log_2 a = 1 \Rightarrow a = 2$ at any point
$(x^2 + 6x + 9)x = 8x + 4$		
$x^3 + 6x^2 + x - 4 = 0$	A1 **5**	Confirm given equation correctly
(b) $x > 0$, otherwise $\log_2 x$ is undefined	B1*	State or imply that log x only defined for $x > 0$
$x = \frac{1}{2}\left(-5 + \sqrt{41}\right)$	B1√dep*	State $x = \frac{1}{2}\left(-5 + \sqrt{41}\right)$ (or x = 0.7) only, following their single positive root in (i)(b)
	2	
	14	